

# College Algebra

LIAL | HORNSBY | SCHNEIDER | DANIELS

TWELFTH EDITION





Available in MyMathLab<sup>®</sup> for Your Course



## Achieve Your Potential

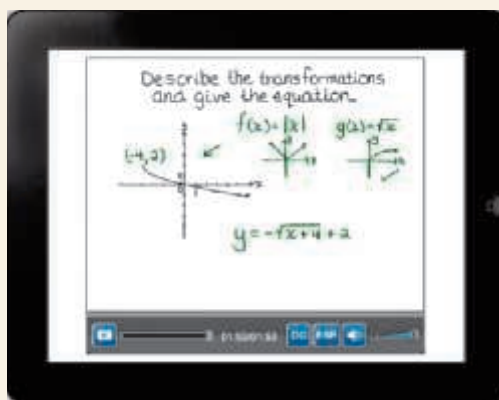
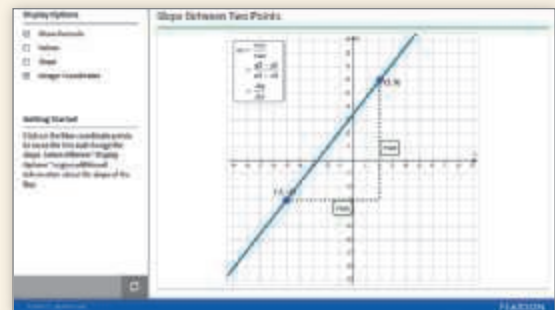
Success in math can make a difference in your life. MyMathLab is a learning experience with resources to help you achieve your potential in this course and beyond. MyMathLab will help you learn the new skills required, and also help you learn the concepts and make connections for future courses and careers.

## Visualization and Conceptual Understanding

These MyMathLab resources will help you think visually and connect the concepts.

### NEW! Guided Visualizations

These engaging interactive figures bring mathematical concepts to life, helping students visualize the concepts through directed explorations and purposeful manipulation. Guided Visualizations are assignable in MyMathLab and encourage active learning, critical thinking, and conceptual learning.



### Video Assessment Exercises

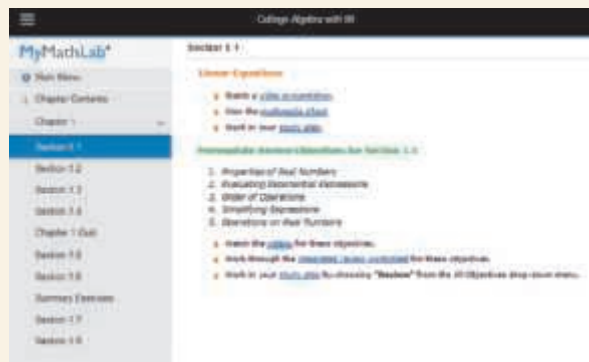
Assignable in MyMathLab, Example Solution Videos present the detailed solution process for every example in the text. Additional Quick Reviews cover definitions and procedures for each section. Assessment exercises check conceptual understanding of the mathematics.

## Preparedness and Study Skills

MyMathLab® gives access to many learning resources which refresh knowledge of topics previously learned. Integrated Review MyMathLab Courses, Getting Ready material and Skills for Success Modules are some of the tools available.

### Integrated Review Courses

This new type of MyMathLab course offers a complete college algebra course with select topics from developmental algebra. Each chapter begins with a *Skills Check* assignment to pinpoint which prerequisite topics, if any, need review. A personalized review homework assignment is automatically generated and provides extra support. Additional review materials (worksheets, videos, and more) are available at the start of each chapter.

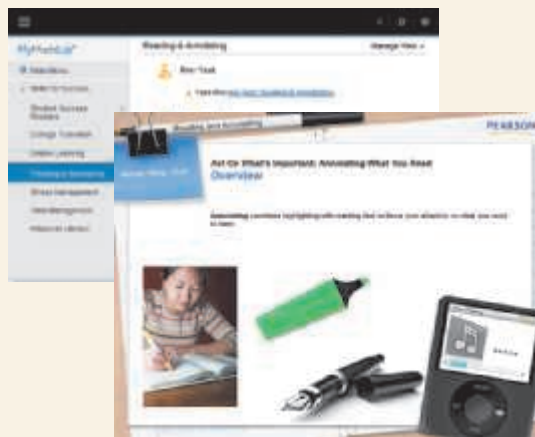


### Getting Ready

Students refresh prerequisite topics through skill review quizzes and personalized homework integrated in MyMathLab. With Getting Ready content in MyMathLab students get just the help they need to be prepared to learn the new material.

### Skills for Success Modules

Skills for Success Modules help foster success in collegiate courses and prepare students for future professions. Topics such as “Time Management,” “Stress Management” and “Financial Literacy” are available within the MyMathLab course.



# College Algebra

**TWELFTH EDITION**

This page intentionally left blank

# College Algebra

**TWELFTH EDITION**

**Margaret L. Lial**

*American River College*

**John Hornsby**

*University of New Orleans*

**David I. Schneider**

*University of Maryland*

**Callie J. Daniels**

*St. Charles Community College*

**PEARSON**

Boston Columbus Indianapolis New York San Francisco  
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montréal Toronto  
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

**To Larry, Jean, Tommy, Mike, and Chuck  
E.J.H.**

**To the memory of my grandparents, Fred and Katherine Cole  
C.J.D.**



This page intentionally left blank

# Contents

*Preface* xiii

*Resources for Success* xviii

## **R** Review of Basic Concepts 1

### **R.1** Sets 2

Basic Definitions ■ Operations on Sets

### **R.2** Real Numbers and Their Properties 9

Sets of Numbers and the Number Line ■ Exponents ■ Order of Operations ■ Properties of Real Numbers ■ Order on the Number Line ■ Absolute Value

### **R.3** Polynomials 24

Rules for Exponents ■ Polynomials ■ Addition and Subtraction ■ Multiplication ■ Division

### **R.4** Factoring Polynomials 36

Factoring Out the Greatest Common Factor ■ Factoring by Grouping ■ Factoring Trinomials ■ Factoring Binomials ■ Factoring by Substitution

### **R.5** Rational Expressions 46

Rational Expressions ■ Lowest Terms of a Rational Expression ■ Multiplication and Division ■ Addition and Subtraction ■ Complex Fractions

### **R.6** Rational Exponents 55

Negative Exponents and the Quotient Rule ■ Rational Exponents ■ Complex Fractions Revisited

### **R.7** Radical Expressions 66

Radical Notation ■ Simplified Radicals ■ Operations with Radicals ■ Rationalizing Denominators

Test Prep 77 ■ Review Exercises 81 ■ Test 85

## **1** Equations and Inequalities 87

### **1.1** Linear Equations 88

Basic Terminology of Equations ■ Linear Equations ■ Identities, Conditional Equations, and Contradictions ■ Solving for a Specified Variable (Literal Equations)

### **1.2** Applications and Modeling with Linear Equations 94

Solving Applied Problems ■ Geometry Problems ■ Motion Problems ■ Mixture Problems ■ Modeling with Linear Equations

### **1.3** Complex Numbers 105

Basic Concepts of Complex Numbers ■ Operations on Complex Numbers

**1.4 Quadratic Equations 113**

The Zero-Factor Property ■ The Square Root Property ■ Completing the Square ■ The Quadratic Formula ■ Solving for a Specified Variable ■ The Discriminant

Chapter 1 Quiz (Sections 1.1-1.4) 123

**1.5 Applications and Modeling with Quadratic Equations 124**

Geometry Problems ■ The Pythagorean Theorem ■ Height of a Projected Object ■ Modeling with Quadratic Equations

**1.6 Other Types of Equations and Applications 136**

Rational Equations ■ Work Rate Problems ■ Equations with Radicals ■ Equations with Rational Exponents ■ Equations Quadratic in Form

Summary Exercises on Solving Equations 149

**1.7 Inequalities 150**

Linear Inequalities ■ Three-Part Inequalities ■ Quadratic Inequalities ■ Rational Inequalities

**1.8 Absolute Value Equations and Inequalities 162**

Basic Concepts ■ Absolute Value Equations ■ Absolute Value Inequalities ■ Special Cases ■ Absolute Value Models for Distance and Tolerance

Test Prep 170 ■ Review Exercises 175 ■ Test 181

**2 Graphs and Functions 183****2.1 Rectangular Coordinates and Graphs 184**

Ordered Pairs ■ The Rectangular Coordinate System ■ The Distance Formula ■ The Midpoint Formula ■ Equations in Two Variables

**2.2 Circles 195**

Center-Radius Form ■ General Form ■ An Application

**2.3 Functions 203**

Relations and Functions ■ Domain and Range ■ Determining Whether Relations Are Functions ■ Function Notation ■ Increasing, Decreasing, and Constant Functions

**2.4 Linear Functions 219**

Basic Concepts of Linear Functions ■ Standard Form  $Ax + By = C$  ■ Slope ■ Average Rate of Change ■ Linear Models

Chapter 2 Quiz (Sections 2.1-2.4) 233

**2.5 Equations of Lines and Linear Models 234**

Point-Slope Form ■ Slope-Intercept Form ■ Vertical and Horizontal Lines ■ Parallel and Perpendicular Lines ■ Modeling Data ■ Graphical Solution of Linear Equations in One Variable

Summary Exercises on Graphs, Circles, Functions, and Equations 247

**2.6 Graphs of Basic Functions 248**

Continuity ■ The Identity, Squaring, and Cubing Functions ■ The Square Root and Cube Root Functions ■ The Absolute Value Function ■ Piecewise-Defined Functions ■ The Relation  $x = y^2$

**2.7 Graphing Techniques 260**

Stretching and Shrinking ■ Reflecting ■ Symmetry ■ Even and Odd Functions ■ Translations

Chapter 2 Quiz (Sections 2.5–2.7) 276

**2.8 Function Operations and Composition 277**

Arithmetic Operations on Functions ■ The Difference Quotient ■ Composition of Functions and Domain

Test Prep 292 ■ Review Exercises 296 ■ Test 301

**3****Polynomial and Rational Functions 303****3.1 Quadratic Functions and Models 304**

Polynomial Functions ■ Quadratic Functions ■ Graphing Techniques ■ Completing the Square ■ The Vertex Formula ■ Quadratic Models

**3.2 Synthetic Division 320**

Synthetic Division ■ Remainder Theorem ■ Potential Zeros of Polynomial Functions

**3.3 Zeros of Polynomial Functions 327**

Factor Theorem ■ Rational Zeros Theorem ■ Number of Zeros ■ Conjugate Zeros Theorem ■ Zeros of a Polynomial Function ■ Descartes' Rule of Signs

**3.4 Polynomial Functions: Graphs, Applications, and Models 339**

Graphs of  $f(x) = ax^n$  ■ Graphs of General Polynomial Functions ■ Behavior at Zeros ■ Turning Points and End Behavior ■ Graphing Techniques ■ Intermediate Value and Boundedness Theorems ■ Approximations of Real Zeros ■ Polynomial Models

Summary Exercises on Polynomial Functions, Zeros, and Graphs 358

**3.5 Rational Functions: Graphs, Applications, and Models 360**

The Reciprocal Function  $f(x) = \frac{1}{x}$  ■ The Function  $f(x) = \frac{1}{x^2}$  ■ Asymptotes ■ Graphing Techniques ■ Rational Models

Chapter 3 Quiz (Sections 3.1–3.5) 381

Summary Exercises on Solving Equations and Inequalities 381

**3.6 Variation 383**

Direct Variation ■ Inverse Variation ■ Combined and Joint Variation

Test Prep 392 ■ Review Exercises 397 ■ Test 402



## 4 Inverse, Exponential, and Logarithmic Functions 405

### 4.1 Inverse Functions 406

One-to-One Functions ■ Inverse Functions ■ Equations of Inverses ■  
An Application of Inverse Functions to Cryptography

### 4.2 Exponential Functions 419

Exponents and Properties ■ Exponential Functions ■ Exponential Equations ■ Compound Interest ■ The Number  $e$  and Continuous Compounding ■ Exponential Models

### 4.3 Logarithmic Functions 435

Logarithms ■ Logarithmic Equations ■ Logarithmic Functions ■  
Properties of Logarithms

Summary Exercises on Inverse, Exponential, and Logarithmic Functions 448

### 4.4 Evaluating Logarithms and the Change-of-Base Theorem 449

Common Logarithms ■ Applications and Models with Common Logarithms ■  
Natural Logarithms ■ Applications and Models with Natural Logarithms ■  
Logarithms with Other Bases

Chapter 4 Quiz (Sections 4.1–4.4) 461

### 4.5 Exponential and Logarithmic Equations 461

Exponential Equations ■ Logarithmic Equations ■ Applications and Models

### 4.6 Applications and Models of Exponential Growth and Decay 473

The Exponential Growth or Decay Function ■ Growth Function Models ■  
Decay Function Models

Summary Exercises on Functions: Domains and Defining Equations 485

Test Prep 488 ■ Review Exercises 491 ■ Test 495

## 5 Systems and Matrices 497

### 5.1 Systems of Linear Equations 498

Linear Systems ■ Substitution Method ■ Elimination Method ■  
Special Systems ■ Application of Systems of Equations ■  
Linear Systems with Three Unknowns (Variables) ■  
Application of Systems to Model Data

### 5.2 Matrix Solution of Linear Systems 516

The Gauss-Jordan Method ■ Special Systems

**5.3 Determinant Solution of Linear Systems 528**

Determinants ■ Cofactors ■  $n \times n$  Determinants ■ Determinant Theorems ■ Cramer's Rule

**5.4 Partial Fractions 541**

Decomposition of Rational Expressions ■ Distinct Linear Factors ■ Repeated Linear Factors ■ Distinct Linear and Quadratic Factors ■ Repeated Quadratic Factors

Chapter 5 Quiz (Sections 5.1–5.4) 548

**5.5 Nonlinear Systems of Equations 549**

Nonlinear Systems with Real Solutions ■ Nonlinear Systems with Nonreal Complex Solutions ■ An Application of Nonlinear Systems

Summary Exercises on Systems of Equations 559

**5.6 Systems of Inequalities and Linear Programming 560**

Linear Inequalities in Two Variables ■ Systems of Inequalities ■ Linear Programming

**5.7 Properties of Matrices 572**

Basic Definitions ■ Matrix Addition ■ Special Matrices ■ Matrix Subtraction ■ Scalar Multiplication ■ Matrix Multiplication ■ An Application of Matrix Algebra

**5.8 Matrix Inverses 586**

Identity Matrices ■ Multiplicative Inverses ■ Solution of Systems Using Inverse Matrices

Test Prep 598 ■ Review Exercises 603 ■ Test 609

**6 Analytic Geometry 611****6.1 Parabolas 612**

Conic Sections ■ Horizontal Parabolas ■ Geometric Definition and Equations of Parabolas ■ An Application of Parabolas

**6.2 Ellipses 621**

Equations and Graphs of Ellipses ■ Translated Ellipses ■ Eccentricity ■ Applications of Ellipses

Chapter 6 Quiz (Sections 6.1–6.2) 633

**6.3 Hyperbolas 633**

Equations and Graphs of Hyperbolas ■ Translated Hyperbolas ■ Eccentricity

**6.4 Summary of the Conic Sections 643**

Characteristics ■ Identifying Conic Sections ■ Geometric Definition of Conic Sections

Test Prep 650 ■ Review Exercises 652 ■ Test 655

## 7 Further Topics in Algebra 657

### 7.1 Sequences and Series 658

Sequences ■ Series and Summation Notation ■ Summation Properties and Rules

### 7.2 Arithmetic Sequences and Series 669

Arithmetic Sequences ■ Arithmetic Series

### 7.3 Geometric Sequences and Series 679

Geometric Sequences ■ Geometric Series ■ Infinite Geometric Series ■ Annuities

Summary Exercises on Sequences and Series 690

### 7.4 The Binomial Theorem 691

A Binomial Expansion Pattern ■ Pascal's Triangle ■  $n$ -Factorial ■ Binomial Coefficients ■ The Binomial Theorem ■  $k$ th Term of a Binomial Expansion

### 7.5 Mathematical Induction 698

Principle of Mathematical Induction ■ Proofs of Statements ■ Generalized Principle of Mathematical Induction ■ Proof of the Binomial Theorem

Chapter 7 Quiz (Sections 7.1–7.5) 705

### 7.6 Basics of Counting Theory 706

Fundamental Principle of Counting ■ Permutations ■ Combinations ■ Characteristics That Distinguish Permutations from Combinations

### 7.7 Basics of Probability 717

Basic Concepts ■ Complements and Venn Diagrams ■ Odds ■ Compound Events ■ Binomial Probability

Test Prep 729 ■ Review Exercises 733 ■ Test 737

*Answers to Selected Exercises* A-1

*Photo Credits* C-1

*Index* I-1

## WELCOME TO THE 12TH EDITION

In the twelfth edition of *College Algebra*, we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyMathLab.

In this edition, we have drawn upon the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability with improved layout of examples, better use of color in displays, and language written with students in mind. Over 300 calculator screenshots have been updated and now provide color displays to enhance students' conceptual understanding. Each homework section now begins with six to ten *Concept Preview* exercises, assignable in MyMathLab, which may be used to ensure students' understanding of vocabulary and basic concepts prior to beginning the regular homework exercises.

Further enhancements include numerous current data examples and exercises that have been updated to reflect current information. Additional real-life exercises have been included to pique student interest; answers to writing exercises have been provided; better consistency has been achieved between the directions that introduce examples and those that introduce the corresponding exercises; and better guidance for rounding of answers has been provided in the exercise sets.

The Lial team believes this to be our best *College Algebra* edition yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional textbooks in this series are as follows.

*Trigonometry*, Eleventh Edition  
*College Algebra and Trigonometry*, Sixth Edition  
*Precalculus*, Sixth Edition

## HIGHLIGHTS OF NEW CONTENT

- In **Chapter R**, more detail has been added to set-builder notation, illustrations of the rules for exponents have been provided, and many exercises have been updated to better match section examples.
- Several new and updated application exercises have been inserted into the **Chapter 1** exercise sets. New objectives have been added to **Section 1.4** outlining the four methods for solving a quadratic equation, along with guidance suggesting when each method may be used efficiently.
- **Chapters 2 and 3** contain numerous new and updated application exercises, along with many updated calculator screenshots that are now provided in color. In response to reviewer suggestions, the discussion on increasing, decreasing, and constant functions in **Section 2.3** has been written to apply to open intervals of the domain.
- In **Chapter 4**, greater emphasis is given to the concept of exponential and logarithmic functions as inverses, there is a new table providing descriptions of the additional properties of exponents, and additional exercises requiring graphing logarithmic functions with translations have been included. There are also many new and updated real-life applications of exponential and logarithmic functions.




- In **Chapter 5**, special attention has been given to finding partial fraction decompositions in **Section 5.4** and to linear programming in **Section 5.6**. Examples have been rewritten to promote student understanding of these difficult topics.
- In **Chapter 6**, greater emphasis is given to analyzing the specific aspects of conic sections, such as finding the equation of the axis of symmetry of a parabola, finding the coordinates of the foci of ellipses and hyperbolas, and finding the equations of the asymptotes of hyperbolas.
- Throughout **Chapter 7**, examples have been carefully updated to ensure that students are able to understand each step of the solutions. Special consideration was given to mathematical induction in **Section 7.5** by providing numerous additional side comments for the steps in the solution of examples in this difficult section.
- For visual learners, numbered **Figure** and **Example** references within the text are set using the same typeface as the figure number itself and bold print for the example. This makes it easier for the students to identify and connect them. We also have increased our use of a “drop down” style, when appropriate, to distinguish between simplifying expressions and solving equations, and we have added many more explanatory side comments. Guided Visualizations, with accompanying exercises and explorations, are now available and assignable in MyMathLab.
- *College Algebra* is widely recognized for the quality of its exercises. In the twelfth edition, nearly 1000 are new or modified, and hundreds present updated real-life data. Furthermore, the MyMathLab course has expanded coverage of all exercise types appearing in the exercise sets, as well as the mid-chapter Quizzes and Cumulative Reviews.

## FEATURES OF THIS TEXT

### SUPPORT FOR LEARNING CONCEPTS



We provide a variety of features to support students' learning of the essential topics of college algebra. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student has understood the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as business, entertainment, sports, biology, astronomy, geology, and environmental studies.

- **Function Boxes** Beginning in Chapter 2, functions provide a unifying theme throughout the text. Special function boxes offer a comprehensive, visual introduction to each type of function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyMathLab.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to promote visual appeal. NEW Guided Visualizations with accompanying exercises and explorations are now available and assignable in MyMathLab.
- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although *this technology is completely optional and can be omitted without loss of continuity*. We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon .
- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.

## SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master college algebra. The extensive exercise sets provide ample opportunity for practice, and the exercise problems increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes understanding of the concepts and reduces the need for rote memorization.

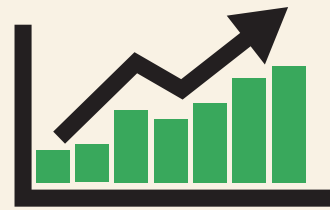
- **NEW Concept Preview** Each exercise set now begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts of each section. These new exercises are assignable in MyMathLab and will provide support especially for hybrid, online, and flipped courses.
- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator problems , and multiple-choice, matching, true/false, and completion exercises. Those marked *Concept Check* focus on conceptual thinking. *Connecting Graphs with Equations* exercises challenge students to write equations that correspond to given graphs.
- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyMathLab.
- **Complete Solutions to Selected Exercises** Complete solutions to all exercises marked  are available in the eText. These are often exercises that extend the skills and concepts presented in the numbered examples.

## SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found within the chapters and at the ends of chapters. Quizzes that are interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter "Test Preps" provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyMathLab.
- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods.
- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms**, a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test**. The Test Prep, Review Exercises, and Chapter Test are assignable in MyMathLab.

# Get the most out of MyMathLab<sup>®</sup>



MyMathLab is the world's leading online resource for teaching and learning mathematics. MyMathLab helps students and instructors improve results, and it provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

## Personalized Support for Students

- MyMathLab comes with many learning resources—eText, animations, videos, and more—all designed to support your students as they progress through their course.
- The Adaptive Study Plan acts as a personal tutor, updating in real time based on student performance to provide personalized recommendations on what to work on next. With the new Companion Study Plan assignments, instructors can now assign the Study Plan as a prerequisite to a test or quiz, helping to guide students through concepts they need to master.
- Personalized Homework enables instructors to create homework assignments tailored to each student's specific needs and focused on the topics they have not yet mastered.

Used by nearly 4 million students each year, the MyMathLab and MyStatLab family of products delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success.



# Resources for Success

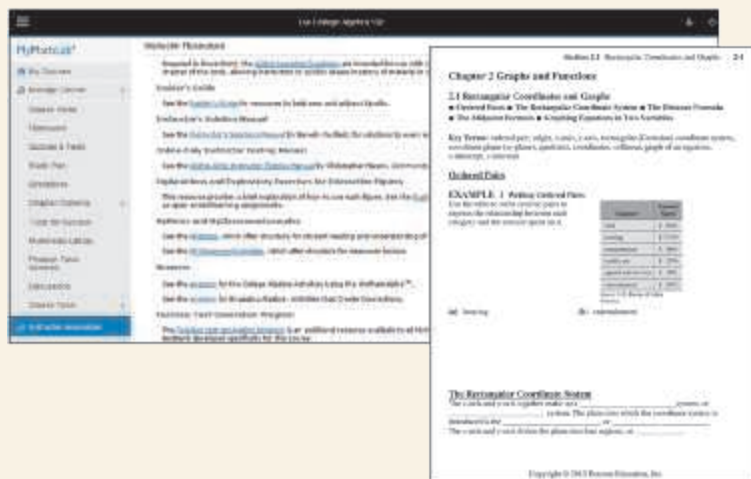
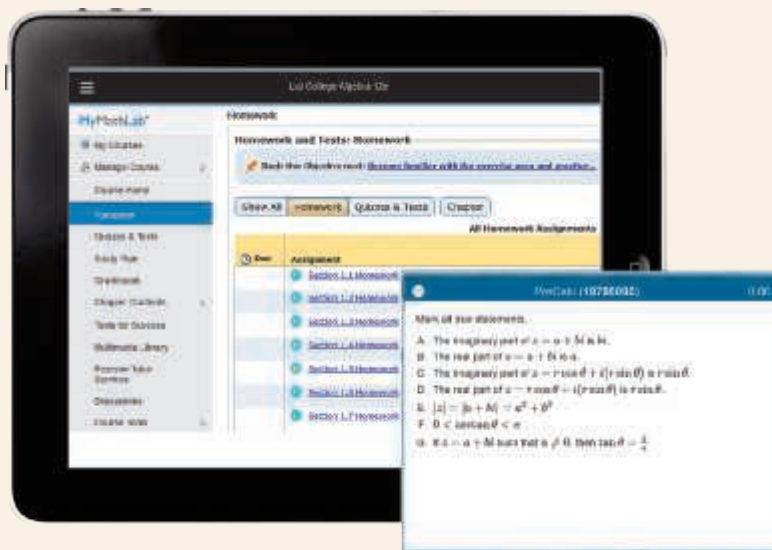
## MyMathLab® Online Course for College Algebra

by Lial, Hornsby, Schneider, and Daniels

MyMathLab delivers proven results in helping individual students succeed. The authors Lial, Hornsby, Schneider, and Daniels have developed specific content in MyMathLab to give students the practice they need to develop a conceptual understanding of College Algebra and the analytical skills necessary for success in mathematics. The MyMathLab features described here support College Algebra students in a variety of classroom formats (traditional, hybrid, and online).

### Concept Preview Exercises

Exercise sets now begin with a group of Concept Preview Exercises, assignable in MyMathLab and also available in Learning Catalytics. These may be used to ensure that students understand the related vocabulary and basic concepts before beginning the regular homework problems. Learning Catalytics is a “bring your own device” system of prebuilt questions designed to enhance student engagement and facilitate assessment.



### MyNotes and MyClassroomExamples

MyNotes provide a note-taking structure for students to use while they read the text or watch the MyMathLab videos. MyClassroom Examples offer structure for notes taken during lecture and are for use with the Classroom Examples found in the Annotated Instructor Edition.

Both sets of notes are available in MyMathLab and can be customized by the instructor.

# Resources for Success

## Student Supplements

### Student's Solutions Manual

By Beverly Fusfield

- Provides detailed solutions to all odd-numbered text exercises

ISBN: 0-13-428268-X & 978-0-13-428268-8

### Video Lectures with Optional Captioning

- Feature Quick Reviews and Example Solutions:
  - Quick Reviews cover key definitions and procedures from each section.
  - Example Solutions walk students through the detailed solution process for every example in the textbook.
- Ideal for distance learning or supplemental instruction at home or on campus
- Include optional text captioning
- Available in MyMathLab®

### MyNotes

- Available in MyMathLab and offer structure for students as they watch videos or read the text
- Include textbook examples along with ample space for students to write solutions and notes
- Include key concepts along with prompts for students to read, write, and reflect on what they have just learned
- Customizable so that instructors can add their own examples or remove examples that are not covered in their courses

### MyClassroomExamples

- Available in MyMathLab and offer structure for classroom lecture
- Include Classroom Examples along with ample space for students to write solutions and notes
- Include key concepts along with fill in the blank opportunities to keep students engaged
- Customizable so that instructors can add their own examples or remove Classroom Examples that are not covered in their courses

## Instructor Supplements

### Annotated Instructor's Edition

- Provides answers in the margins to almost all text exercises, as well as helpful Teaching Tips and Classroom Examples
- Includes sample homework assignments indicated by exercise numbers underlined in blue within each end-of-section exercise set
- Sample homework exercises assignable in MyMathLab

ISBN: 0-13-421765-9 & 978-0-13-421765-9

### Online Instructor's Solutions Manual

By Beverly Fusfield

- Provides complete solutions to all text exercises
- Available in MyMathLab or downloadable from Pearson Education's online catalog

### Online Instructor's Testing Manual

By David Atwood

- Includes diagnostic pretests, chapter tests, final exams, and additional test items, grouped by section, with answers provided
- Available in MyMathLab or downloadable from Pearson Education's online catalog

### TestGen®

- Enables instructors to build, edit, print, and administer tests
- Features a computerized bank of questions developed to cover all text objectives
- Available in MyMathLab or downloadable from Pearson Education's online catalog

### Online PowerPoint Presentation and Classroom Example PowerPoints

- Written and designed specifically for this text
- Include figures and examples from the text
- Provide Classroom Example PowerPoints that include full worked-out solutions to all Classroom Examples
- Available in MyMathLab or downloadable from Pearson Education's online catalog

## ACKNOWLEDGMENTS

We wish to thank the following individuals who provided valuable input into this edition of the text.

**Barbara Aramenta** – Pima Community College  
**Robert Bates** – Honolulu Community College  
**Troy Brachey** – Tennessee Tech University  
**Hugh Cornell** – University of North Florida  
**John E. Daniels** – Central Michigan University  
**Dan Fahringer** – HACC Harrisburg  
**Doug Grenier** – Rogers State University  
**Mary Hill** – College of DuPage  
**Keith Hubbard** – Stephen Austin State University  
**Christine Janowiak** – Arapahoe Community College  
**Tarcia Jones** – Austin Community College, Rio Grande  
**Rene Lumampao** – Austin Community College  
**Rosana Maldonado** – South Texas Community College, Pecan  
**Nilay S. Manzagol** – Georgia State University  
**Marianna McClymonds** – Phoenix College  
**Randy Nichols** – Delta College  
**Preeti Parikh** – SUNY Maritime  
**Deanna Robinson-Briedel** – McLennan Community College  
**Sutandra Sarkar** – Georgia State University  
**Patty Schovanec** – Texas Tech University  
**Jimmy Vincente** – El Centro College  
**Deanna M. Welsch** – Illinois Central College  
**Amanda Wheeler** – Amarillo College  
**Li Zhou** – Polk State College

Our sincere thanks to those individuals at Pearson Education who have supported us throughout this revision: Anne Kelly, Christine O'Brien, Joe Vetere, and Danielle Simbajon. Terry McGinnis continues to provide behind-the-scenes guidance for both content and production. We have come to rely on her expertise during all phases of the revision process. Marilyn Dwyer of Cenveo® Publishing Services, with the assistance of Carol Merrigan, provided excellent production work. Special thanks go out to Paul Lorzak and John Morin for their excellent accuracy-checking. We thank Lucie Haskins, who provided an accurate index, and Jack Hornsby, who provided assistance in creating calculator screens, researching data updates, and proofreading. We appreciate the valuable suggestions for Chapter 5 that Mary Hill of *College of Dupage* made during our meeting with her in March 2010.

As an author team, we are committed to providing the best possible college algebra course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to [math@pearson.com](mailto:math@pearson.com).

*Margaret L. Lial*  
*John Hornsby*  
*David I. Schneider*  
*Callie J. Daniels*

# College Algebra

**TWELFTH EDITION**



This page intentionally left blank

# R

## Review of Basic Concepts



Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

- R.1** Sets
- R.2** Real Numbers and Their Properties
- R.3** Polynomials
- R.4** Factoring Polynomials
- R.5** Rational Expressions
- R.6** Rational Exponents
- R.7** Radical Expressions



**SOLUTION**

- (a) The set is finite, because the process of counting its elements 7, 8, 9, 10, 11, 12, 13, and 14 comes to an end. The number 10 belongs to the set.

$$10 \in \{7, 8, 9, \dots, 14\}$$

- (b) The set is infinite, because the ellipsis points indicate that the pattern continues indefinitely. In this case,

$$10 \notin \left\{1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots\right\}.$$

- (c) Between any two distinct natural numbers there are infinitely many fractions, so this set is infinite. The number 10 is not an element.
- (d) There is only one natural number between 9 and 11, namely 10. So the set is finite, and 10 is an element.

✔ **Now Try Exercises 11, 13, 15, and 17.**

**EXAMPLE 2 Listing the Elements of a Set**

Use set notation, and list all the elements of each set.

- (a)  $\{x \mid x \text{ is a natural number less than } 5\}$
- (b)  $\{x \mid x \text{ is a natural number greater than } 7 \text{ and less than } 14\}$

**SOLUTION**

- (a) The natural numbers less than 5 form the set  $\{1, 2, 3, 4\}$ .
- (b) This is the set  $\{8, 9, 10, 11, 12, 13\}$ .

✔ **Now Try Exercise 25.**

When we are discussing a particular situation or problem, the **universal set** (whether expressed or implied) contains all the elements included in the discussion. The letter  $U$  is used to represent the universal set. The **null set**, or **empty set**, is the set containing no elements. We write the null set by either using the special symbol  $\emptyset$ , or else writing set braces enclosing no elements,  $\{\}$ .

---

**CAUTION** Do not combine these symbols.  $\{\emptyset\}$  is *not the null set*. It is the set containing the symbol  $\emptyset$ .

---

Every element of the set  $S = \{1, 2, 3, 4\}$  is a natural number.  $S$  is an example of a *subset* of the set  $N$  of natural numbers. This relationship is written using the symbol  $\subseteq$ .

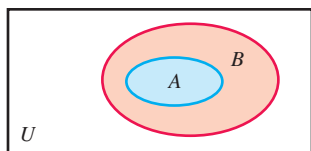
$$S \subseteq N$$

By definition, set  $A$  is a **subset** of set  $B$  if every element of set  $A$  is also an element of set  $B$ . For example, if  $A = \{2, 5, 9\}$  and  $B = \{2, 3, 5, 6, 9, 10\}$ , then  $A \subseteq B$ . However, there are some elements of  $B$  that are not in  $A$ , so  $B$  is not a subset of  $A$ . This relationship is written using the symbol  $\not\subseteq$ .

$$B \not\subseteq A$$

Every set is a subset of itself. Also,  $\emptyset$  is a subset of every set.

**If  $A$  is any set, then  $A \subseteq A$  and  $\emptyset \subseteq A$ .**



$$A \subseteq B$$

Figure 1

**Figure 1** shows a set  $A$  that is a subset of set  $B$ . The rectangle in the drawing represents the universal set  $U$ . Such a diagram is a **Venn diagram**.

Two sets  $A$  and  $B$  are equal whenever  $A \subseteq B$  and  $B \subseteq A$ . Equivalently,  $A = B$  if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

is true because both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\}$$

because the set  $\{0, 1, 2, 3\}$  contains the element 0, which is not an element of  $\{1, 2, 3\}$ .

### EXAMPLE 3 Examining Subset Relationships

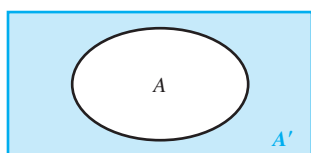
Let  $U = \{1, 3, 5, 7, 9, 11, 13\}$ ,  $A = \{1, 3, 5, 7, 9, 11\}$ ,  $B = \{1, 3, 7, 9\}$ ,  $C = \{3, 9, 11\}$ , and  $D = \{1, 9\}$ . Determine whether each statement is *true* or *false*.

- (a)  $D \subseteq B$       (b)  $B \subseteq D$       (c)  $C \not\subseteq A$       (d)  $U = A$

#### SOLUTION

- (a) All elements of  $D$ , namely 1 and 9, are also elements of  $B$ , so  $D$  is a subset of  $B$ , and  $D \subseteq B$  is true.
- (b) There is at least one element of  $B$  (for example, 3) that is not an element of  $D$ , so  $B$  is *not* a subset of  $D$ . Thus,  $B \subseteq D$  is false.
- (c)  $C$  is a subset of  $A$ , because every element of  $C$  is also an element of  $A$ . Thus,  $C \subseteq A$  is true, and as a result,  $C \not\subseteq A$  is false.
- (d)  $U$  contains the element 13, but  $A$  does not. Therefore,  $U = A$  is false.

✓ **Now Try Exercises 53, 55, 63, and 65.**



$$A'$$

Figure 2

**Operations on Sets** Given a set  $A$  and a universal set  $U$ , the set of all elements of  $U$  that do *not* belong to set  $A$  is the **complement** of set  $A$ . For example, if set  $A$  is the set of all students in a class 30 years old or older, and set  $U$  is the set of all students in the class, then the complement of  $A$  would be the set of all students in the class younger than age 30.

The complement of set  $A$  is written  $A'$  (read “**A-prime**”). The Venn diagram in **Figure 2** shows a set  $A$ . Its complement,  $A'$ , is in color. Using set-builder notation, the complement of set  $A$  is described as follows.

$$A' = \{x \mid x \in U, x \notin A\}$$

### EXAMPLE 4 Finding Complements of Sets

Let  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 3, 5, 7\}$ , and  $B = \{3, 4, 6\}$ . Find each set.

- (a)  $A'$       (b)  $B'$       (c)  $\emptyset'$       (d)  $U'$

#### SOLUTION

- (a) Set  $A'$  contains the elements of  $U$  that are not in  $A$ . Thus,  $A' = \{2, 4, 6\}$ .

- (b)  $B' = \{1, 2, 5, 7\}$       (c)  $\emptyset' = U$       (d)  $U' = \emptyset$

✓ **Now Try Exercise 89.**

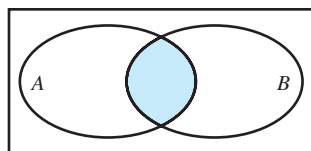
 $A \cap B$ 

Figure 3

Given two sets  $A$  and  $B$ , the set of all elements belonging both to set  $A$  *and* to set  $B$  is the **intersection** of the two sets, written  $A \cap B$ . For example, if  $A = \{1, 2, 4, 5, 7\}$  and  $B = \{2, 4, 5, 7, 9, 11\}$ , then we have the following.

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}$$

The Venn diagram in **Figure 3** shows two sets  $A$  and  $B$ . Their intersection,  $A \cap B$ , is in color. Using set-builder notation, the intersection of sets  $A$  and  $B$  is described as follows.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Two sets that have no elements in common are **disjoint sets**. If  $A$  and  $B$  are any two disjoint sets, then  $A \cap B = \emptyset$ . For example, there are no elements common to both  $\{50, 51, 54\}$  and  $\{52, 53, 55, 56\}$ , so these two sets are disjoint.

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset$$

### EXAMPLE 5 Finding Intersections of Two Sets

Find each of the following. Identify any disjoint sets.

(a)  $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$

(b)  $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

(c)  $\{1, 3, 5\} \cap \{2, 4, 6\}$

#### SOLUTION

(a)  $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$

The elements 15 and 25 are the only ones belonging to both sets.

(b)  $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

(c)  $\{1, 3, 5\} \cap \{2, 4, 6\} = \emptyset$  Disjoint sets

✓ Now Try Exercises 69, 75, and 85.

The set of all elements belonging to set  $A$  *or* to set  $B$  (or to both) is the **union** of the two sets, written  $A \cup B$ . For example, if  $A = \{1, 3, 5\}$  and  $B = \{3, 5, 7, 9\}$ , then we have the following.

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$$

The Venn diagram in **Figure 4** shows two sets  $A$  and  $B$ . Their union,  $A \cup B$ , is in color.

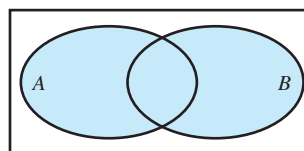
 $A \cup B$ 

Figure 4

Using set-builder notation, the union of sets  $A$  and  $B$  is described as follows.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$